region reduces to a problem of the form (3.2), where the conditions as $Y \to -\infty$ are replaced by the conditions at the trail axis of symmetry $\Psi = \Psi_{YY} = 0$ when $Y = 0$.

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A THREE-DIMENSIONAL HYPERSONIC VISCOUS SHOCK LAYER IN TWO-PHASE FLOW*

S.V. PEIGIN

A three-dimensional hypersonic flow of visous gas containing solid or liquid deformable particles past smooth blunt bodies with permeable surfaces, is considered. A numerical solution is obtained near the stagnation point of double curvature for a wide range of values of the Reynolds number, sizes and compositions of the particles, shape of the body and the injection (suction) parameters. Characteristic velocity and temperature profiles across the shock layer are given for each phase, and also the dependence of the separation, friction and heat exchange coefficients at the body surface on the Reynolds number and other defining parameters of the problem. It is shown that the presence of particles in the flow leads, other conditions being equal, to a reduction in the separation of the shock wave. The asymptotic behaviour of the equations of the three-dimensional two-phase hypersonic shock-layer is analysed for the limiting case of small particles. It is shown that in this case the flow separates into two layers; equations are given for the principal terms of the expansions in each layer, and boundary conditions are given following from the conditions for matching the solutions in adjacent regions. An analytic solution of the problem in the approximation of two inviscid layers separated by a contact surface is obtained for the layer adjacent to the body near the stagnation point for large Reynolds numbers and strong injection.

The motion of heterogeneous particles in plane or axisymmetric shock layers was studied earlier in /1/, in the inviscid formulation and assuming that the effect of the particles on the gas-dynamic parameters is small. A numerical solution of the problem of a supersonic, inviscid two-phase flow past a sphere was obtained in /2—4/. Homogeneous gas flow in a viscous, hypersonic three-dimensional shock layer near the stagnation point was studied in /5/.

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1. Let us consider a three-dimensional hypersonic flow of gas containing spherical particles of radius a and density ρ_s° past a blunt body. Assuming that the conditions of validity of the model of uniformity of the discrete phase hold, we shall study the flow in question using the equations of the mechanics of a two-velocity, two-temperature continuum /6/.

We assume that the Brownian motion of the particles and their interaction are not taken into account, and the volume density of the particles can be neglected.

Let us choose the curvilinear coordinate system $\{x^i\}$ as follows. Let $x^3 = \text{const}$ be the equations of a family of surfaces parallel to the body surface $x^3 = 0$; x^1, x^2 are chosen at the surface. The equations of a three-dimensional hypersonic two-phase viscous shock layer have the following dimensionless form in the $\{x^i\}$-coordinate system:

$$\frac{\partial}{\partial x^i}\left(\rho u^i \sqrt{\frac{g}{g_{(ii)}}}\right) = 0 \tag{1.1}$$

$$\rho\left(Du^\alpha + A^\alpha_{\beta\gamma}u^\beta u^\gamma\right) = -\frac{2\varepsilon}{1+\varepsilon}\sqrt{g_{(\beta\beta)}}\, g^{\alpha\beta}\frac{\partial P}{\partial x^\beta} -$$
$$\varepsilon\mu\beta\varphi\rho_s\left(u^\alpha - u_s^\alpha\right) + \frac{\partial}{\partial x^3}\left(\frac{\mu}{K}\frac{\partial u^\alpha}{\partial x^3}\right)$$
$$\rho A^3_{\alpha\gamma}u^\alpha u^\gamma + \varepsilon^2\mu\beta\varphi\rho_s\left(u^3 - u_s^3\right) = -\frac{2}{1+\varepsilon}\frac{\partial P}{\partial x^3}$$
$$\rho DT = \frac{2\varepsilon}{1+\varepsilon}\frac{u^\alpha}{\sqrt{g_{(\alpha\alpha)}}}\frac{\partial P}{\partial x^\alpha} + \frac{\mu}{K}\Psi_{\alpha\beta}\frac{\partial u^\alpha}{\partial x^3}\frac{\partial u^\beta}{\partial x^3} +$$
$$\varepsilon\mu\beta\rho_s\varphi\left[\Psi_{\alpha\beta}\left(u^\alpha - u_s^\alpha\right)\left(u^\beta - u_s^\beta\right) + \varepsilon^2\left(u^3 - u_s^3\right)^2\right] -$$
$$\frac{2\,\mathrm{Nu}\,\mu\beta\rho_s\varepsilon}{3\sigma}\left(T - T_s\right) + \frac{\partial}{\partial x^3}\left(\frac{\mu}{K\sigma}\frac{\partial T}{\partial x^3}\right)$$
$$\frac{\partial}{\partial x^i}\rho_s u_s^i\sqrt{\frac{g}{g_{(ii)}}}\Bigg) = 0, \quad P = \rho T, \quad \mu = T^\omega$$
$$D_s u_s^\alpha + A^\alpha_{\beta\gamma}u_s^\beta u_s^\gamma + 2\varepsilon A^\alpha_{3\beta}u_s^\beta u_s^3 = \mu\beta\varphi\left(u^\alpha - u_s^\alpha\right)$$
$$D_s u_s^3 + \varepsilon^{-1}A^3_{\alpha\beta}u_s^\alpha u_s^\beta = \mu\beta\varphi\left(u^3 - u_s^3\right)$$
$$D_s T_s = \frac{2\mu\beta\alpha\,\mathrm{Nu}}{3\sigma}\left(T - T_s\right), \quad D \equiv \frac{u^\alpha}{\sqrt{g_{(\alpha\alpha)}}}\frac{\partial}{\partial x^\alpha} + u^3\frac{\partial}{\partial x^3}$$
$$K = \varepsilon\,\mathrm{Re}, \quad \beta = \frac{9R^2\rho_\infty}{2a^2\rho_s^0\,\mathrm{Re}}, \quad \mathrm{Re} = \frac{\rho_\infty V_\infty R}{\mu_0}, \quad \gamma = \frac{c_p}{c_v}$$
$$\varepsilon = \frac{\gamma - 1}{\gamma + 1}, \quad \alpha = \frac{c_p}{c}, \quad \Psi_{\alpha\beta} = \frac{g_{\alpha\beta}}{\sqrt{g_{(\alpha\alpha)}g_{(\beta\beta)}}}$$
$$\mu_0 = \mu(T_0), \quad T_0 = T_\infty(\gamma - 1)M_\infty^2, \quad g_{\alpha\beta} = a_{\alpha\beta}$$

Here pairs of like indices denote summation, except for the pairs in the round brackets which are not summed. The Latin indices take the values 1, 2, 3 and the Greek indices 1, 2; $V_\infty u^\alpha$, $V_\infty \varepsilon u^3$ are the physical components of the velocity vector in the directions x^α, x^3; $P_\infty\varepsilon^{-1}(T_0/T_\infty)P$, $\varepsilon^{-1}\rho_\infty\rho$, $T_0 T$, $\mu_0\mu$, λ, $c_p = \text{const}$ are the pressure, denisty, temperature, the coefficients of viscosity, thermal conductivity and heat capacity of the carrier phase respectively; $\rho_\infty\rho_s$ is the particle "gas" density, c is the heat capacity of the material of the particles, $\sigma =$ const is the Pradtl number, $a_{\alpha\beta}$, $b_{\alpha\beta}$ are the covariant components of the tensors defining the first and second quadratic form of the surface, and the coefficients A_{jk}^i depend in a known manner on $a_{\alpha\beta}$, $b_{\alpha\beta}$ and are given in /5/.

All linear dimensions are referred to the characteristic dimension R, and the normal x^3 coordinate to εR. Here and henceforth the indices s, ∞, w will refer to the particle gas parameters at infinity and at the surface of the streamlined body respectively. We also introduce the function φ defining the difference between the law of particle resistance and Stokes Law ($\varphi = c_x\,\mathrm{Re}_s/24$, $\mathrm{Re}_s = 2a\rho|\mathbf{V} - \mathbf{V}_s|/\mu_0$). In the course of actual computations the functions $\varphi = \varphi(\mathrm{Re}, \mathrm{M})$, $\mathrm{Nu} = \mathrm{Nu}(\mathrm{Re}, \mathrm{M})$ describing the law of phase interaction were determined using the results obtained in /7/. We note that in the special case of Stokes interaction $\varphi \equiv 1$, $\mathrm{Nu} \equiv 1$.

Equations (1.1) were obtained from the Navier-Stokes equations for a two-phase system /8/ written in the $\{x^i\}$-coordinate system, in which we assumed that $\varepsilon \to 0$, $\mathrm{Re}^{-1} \to 0$, while the product $K = \varepsilon\,\mathrm{Re}$ was of the order of unity. The terms with longitudinal pressure gradient are retained in (1.1), since at large Reynolds numbers they play a major part in the layer surrounding the body surface.

In formulating the boundary conditions at the shock wave $x^3 = x_e^3(x^1, x^2)$ we shall assume that the particles pass through the shock wave without changing their compositiom and, that the generalized Rankine-Hugoniot relations written in the hypersonic approximation hold for the gas. We also assume that the particles in the oncoming flow are in thermal and dynamic equilibrium with the gas

$$(u^\alpha - u_\infty^\alpha) u_\infty^3 = \frac{\mu}{K}\frac{\partial u^\alpha}{\partial x^3}, \quad \left(u^3 - \frac{u^\alpha}{\sqrt{g_{(\alpha\alpha)}}}\frac{\partial x_e^3}{\partial x^\alpha}\right) = \frac{u_\infty^3}{\rho} \tag{1.2}$$

$$u_\infty^3\left[T - \frac{1}{2}(u_\infty^3)^2 - \frac{1}{2}\Psi_{\alpha\beta}(u^\alpha - u_\infty^\alpha)(u^\beta - u_\infty^\beta)\right] = \frac{\mu}{K\sigma}\frac{\partial T}{\partial x^3}$$

$$P = \frac{1}{2}(1+\varepsilon)(u_\infty^3)^2, \quad u_s^i = u_\infty^i, \quad \rho_s = \frac{\rho_{s\infty}}{\rho_\infty} = \delta,$$

$$T_s^{-1} = (\gamma - 1) M_\infty^2$$

When $K \to \infty$, which corresponds to large Reynolds numbers, conditions (1.2) become the usual Rankine-Hugoniot conditions for a two-phase medium written in the thin layer approximation /6/. In establishing the boundary conditions at the body surface we assume that the particles reflected from the body can be disregarded. We write the following boundary conditions for the carrier phase on the streamlined surface, with the slippage rate and temperature jump taken into account /9/:

$$u^\alpha = \frac{2-\vartheta}{\vartheta}\sqrt{\frac{\pi\gamma}{2(\gamma+1)}}\frac{\sqrt{\varepsilon}\,\mu}{\rho K\sqrt{T}}\frac{\partial u^\alpha}{\partial x^3}, \quad \rho u^3 = G(x^1, x^2) \tag{1.3}$$

$$T = T_w + \frac{2-\nu}{\nu}\sqrt{\frac{\pi\gamma}{2(\gamma+1)}}\frac{\sqrt{\varepsilon}\,\mu}{\sigma\rho K\sqrt{T}}\frac{2\gamma}{(\gamma+1)}\frac{\partial T}{\partial x^3}$$

Here ϑ is the diffuse reflection coefficient and ν is the accommodation coefficient, both assumed equal to unity during the actual computations, and $G(x^1, x^2)$ is a given function. We note that from (1.3) it follows that the slippage rate and temperature jump are of the order of $\varepsilon^{1/2}K^{-1}$, therefore at low Reynolds numbers the effect in question has a finite influence on the characteristic parameters of the flow. The effect can be disregarded when $\mathrm{Re} \gg 1$.

2. To solve the initial problem numerically, we change in (1.1) to new dependent and independent variables according to the formulas

$$\xi^\alpha = x^\alpha, \quad \zeta = \frac{1}{\Delta}\int_0^{x^3}\rho\sqrt{g}\,dx^3, \quad \Delta = \int_0^{x_e^3}\rho\sqrt{g}\,dx^3 \tag{2.1}$$

$$u'^\alpha = \frac{u^\alpha}{u_*^\alpha} = \frac{\partial f_\alpha}{\partial\zeta}; \quad T = T_*(\xi^1, \xi^2)\theta, \quad u_s^\alpha = u_*^\alpha u_s'^\alpha$$

$$\rho\sqrt{g}\,u^3 = -\frac{\partial}{\partial\xi^\alpha}[\Psi_*^{(\alpha)}(\xi^1, \xi^2) f_\alpha] - \Delta f_\alpha{}^*\frac{\partial f_\alpha}{\partial\zeta}\frac{\partial\varsigma}{\partial x^\alpha}$$

$$\Psi_*{}^\alpha = \Delta f_\alpha{}^* = \Delta\frac{u_*^\alpha}{\sqrt{g_{(\alpha\alpha)}}}, \quad T_s = T_*\theta_s, \quad l = \frac{\mu\rho g}{K\Delta^2}$$

$$v = u^3, \quad v_s = u_s^3, \quad m = \frac{\Delta}{v_s\rho}, \quad \mu = \theta^\omega$$

and we shall discuss the choice of the functions $u_*^\alpha(\xi^1, \xi^2), T_*(\xi^1, \xi^2)$ later.

Let us now specify in more detail the choice of the coordinates $\{x^\alpha\}$ at the body surface. We shall consider a Cartesian $\{y^i\}$ coordinate system with origin at the point of highest pressure on the body, in which the direction of the y^3-axis coincides with the velocity vector of the oncoming flow and the y^1 and y^2 axes are directed along the principal directions of the surface at the stagnation point. Suppose further that $y^3 = f(y^1, y^2)$ is the equation of the surface of the streamlined body. Parametrizing the body surface in the form $x^\alpha = y^\alpha$, we obtain

$$g_{\alpha\alpha} = 1 + q_\alpha^2, \quad g_{12} = q_1 q_2, \quad b_{\alpha\beta} = -\frac{r_{\alpha\beta}}{\sqrt{g}}, \quad q_\alpha = \frac{\partial f}{\partial x^\alpha} \tag{2.2}$$

$$r_{\alpha\beta} = \frac{\partial^2 f}{\partial x^\alpha \partial x^\beta}, \quad u_\infty^\alpha = \frac{\sqrt{g_{(\alpha\alpha)}}}{g}q_\alpha, \quad u_\infty^3 = -\frac{1}{\sqrt{g}}$$

Let us further consider the flow near the stagnation point. By virtue of the choice of the coordinate system $\{y^i\}$ we can write the equation describing the surface of the streamlined body with an accuracy of $O((y^1)^2 + (y^2)^2)$ in the form $2y^3 = (y^1)^2 + k(y^2)^2$ where $k = R_1/R \leqslant 1$, R, R_1 are the radii of principal curvatures of the body surface at the stagnation point. Let us now write $u_*^\alpha = u_\infty^\alpha$, $T_* = {}^1/_2(u_\infty^3)^2$.

Using (2.1) and (2.2) and developing the singularities appearing in the coefficients we obtain the equations of a hypersonic, viscous, two-phase shock layer near the stagnation point with double curvature (the primes are omitted)

$$\frac{\partial}{\partial\zeta}\left(l\frac{\partial u^\alpha}{\partial\zeta}\right) + (f_1 + kf_2)\frac{\partial u^\alpha}{\partial\zeta} = d_{(\alpha)}(u^\alpha)^2 + \tag{2.3}$$

$$\frac{2\varepsilon P_\alpha}{(1+\varepsilon)\rho} + \varepsilon\beta\mu\varphi\frac{\rho_s}{\rho}(u^\alpha - u_s^\alpha)$$

$$\frac{\partial}{\partial\zeta}\left(\frac{l}{\sigma}\frac{\partial\theta}{\partial\zeta}\right)+(f_1+kf_2)\frac{\partial\theta}{\partial\zeta}=\frac{2\,\mathrm{Nu}\,\beta\mu\varepsilon\rho_s}{3\sigma\rho}(\theta-\theta_s)-$$
$$\frac{2\rho_s}{\rho}\beta\varphi\mu\varepsilon^3(v-v_s)^2,\quad \frac{\partial P}{\partial\zeta}=\Delta(1+\varepsilon)\mu\beta\varphi\varepsilon^2\frac{\rho_s}{2\rho}(v_s-v)$$
$$P=\frac{\rho\theta}{2},\quad \rho v=-\Delta(f_1+kf_2),\quad \frac{\partial P_\alpha}{\partial\zeta}=d_{(\alpha)}(1+\varepsilon)\times$$
$$\left\{\Delta d_{(\alpha)}\left[(u^\alpha)^2+\varepsilon\frac{\rho_s}{\rho}(u_s{}^\alpha)^2\right]+\varepsilon^2\left[\Delta v_s u_s{}^\alpha d_{(\alpha)}\frac{\rho_s}{\rho}+\rho_s v_s\frac{\partial v_s}{\partial\zeta}\right]\right\}$$
$$\frac{\partial u_s{}^\alpha}{\partial\zeta}=m\left[\beta\mu\varphi(u^\alpha-u_s{}^\alpha)-d_{(\alpha)}(u_s{}^\alpha)^2\right]$$
$$\frac{d\theta_s}{\partial\zeta}=\frac{2m\,\mathrm{Nu}\,\alpha\beta\mu}{3\sigma}(\theta-\theta_s),\quad \frac{\partial v_s}{\partial\zeta}=m\beta\mu\varphi(v-v_s)$$
$$\frac{\partial\ln|\rho_s v_s|}{\partial\varsigma}=-m(u_s{}^1+d_2u_s{}^2),\quad P_\alpha=(\xi^{(\alpha)}d_{(\alpha)})^{-1}\frac{\partial P}{\partial\xi^\alpha}$$
$$d_1=1,\quad d_2=k,\quad u\equiv\frac{\partial f_1}{\partial\zeta},\quad w\equiv\frac{\partial f_2}{\partial\zeta}$$

The equations for determining P_α are obtained from the third equation of (1.1) acted upon by the operator $(\xi^{(\alpha)}d_{(\alpha)})^{-1}\partial/\partial\xi^\alpha$. The boundary conditions (1.2), (1.3) take the following form in the new variables

$$\zeta=1,\quad l\Delta\frac{\partial u^\alpha}{\partial\zeta}+u^\alpha=1 \tag{2.4}$$
$$\frac{l\Delta}{\sigma}\frac{\partial\theta}{\partial\zeta}+\theta=1,\quad (f_1+kf_2)\Delta=1,\quad P=\frac{1+\varepsilon}{2}$$
$$P_\alpha=-d_\alpha(1+\varepsilon),\ u_s{}^\alpha=1,\ \theta_s=2T_\infty T_0{}^{-1},\quad \rho_s=\delta,\ v_s=-\varepsilon^{-1}$$
$$\zeta=0,\quad \Delta(f_1+kf_2)=-G \tag{2.5}$$
$$u^\alpha=\sqrt{\frac{\pi\gamma}{\theta_w(\gamma+1)}}\sqrt{\varepsilon}\,l\Delta\frac{\partial u^\alpha}{\partial\zeta}$$
$$\theta=\theta_w+2\sqrt{\frac{\varepsilon\pi}{\theta_w}}\left(\frac{\gamma}{\gamma+1}\right)^{3/2}\frac{l\Delta}{\sigma}\frac{\partial\theta}{\partial\zeta}$$

Equations (2.3) and boundary conditions (2.4), (2.5) were solved numerically using an implicit finite difference scheme /10/ of higher order of approximation. To increase the computational accuracy at large Reynolds numbers and large values of the parameter β, the numerical mesh was compressed near the body surface and the shock wave respectively.

The defining parameters of the problem were varied within the following limits: $\varepsilon=0.1$; $\omega=0.5$; $-0.25\leqslant G\leqslant 0.25$; $5\leqslant \mathrm{Re}\leqslant 5\cdot10^6$; $0\leqslant\delta\leqslant 1$; $0.5\leqslant\alpha\leqslant 2$; $0.05\leqslant\theta_w\leqslant 0.5$; $0\leqslant\beta\leqslant 10^3$; $0.01\leqslant k\leqslant 1$, and some of the results are shown in Figs.1—3.

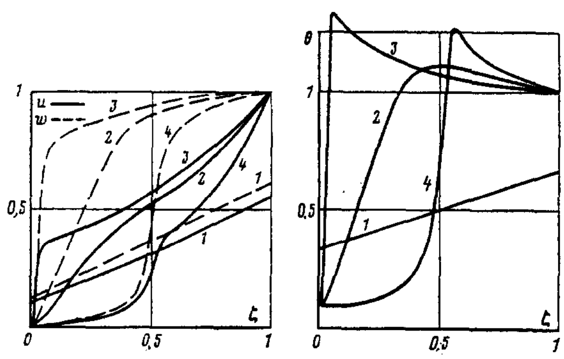

Fig.1

Fig.1 depicts the characteristic profiles u, w, θ across the shock layer for $G=0$; $k=0,1$; $\alpha=0.5$; $\beta=1$; $\delta=1$ at various Reynolds numbers ($\mathrm{Re}=5$, $5\cdot10^2, 5\cdot10^4$, lines 1—3 respectively). We see that a thin boundary layer forms at the body surface as the Reynolds number increases. It should also be of interest that at large Reynolds numbers and small β (fairly large particles) a gas temperature relaxation layer is clearly delineated around the body. Comparing the values of the heat exchange coefficients computed using boundary layer theory with those obtained from the solution of the viscous shock layer equations, we find that they may differ considerably from each other even at large Reynolds numbers. This is explained by the fact that the gas temperature profile obtained from the solution of the external problem (the equations of an inviscid shock layer) has an infinite derivative in the transverse coordinate at the body surface. Because of this the correct construction of the asymptotic formulas for the equations of a viscous, two-phase shock layer requires, for large Reynolds numbers, that the viscous-inviscid interaction, which is not normally taken into account when formulating the problem within the framework of the first approximation of the boundary layer theory, should be taken into account. We also note that the relaxation layer, as shown by the computations, exists even when the particle concentration is fairly low, and this may result in the particles exerting a finite influence on the heat exchange characteristics.

Fig.2 shows the dependence on β of the profiles θ (solid lines) and θ_s (dashed lines) across the shock layer for $\mathrm{Re} = 500$; $k = 0.1$; $\alpha = 0.5$; $\delta = 0.25$. Here $\beta = 1, 8, 130$ correspond to lines *1–3* respectively, and the dot-dash line refers to the profile θ for the limiting case of fine particles. When $\beta \geqslant 150$, the profiles of the carrier phase characteristics coincide with the limit solution practically over the whole shock layer (see Sect.3). The computations have also shown that increasing the parameter β leads, other conditions being equal to reduced separation of the shock wave. This result was obtained earlier in /2/ for plane and axisymmetric flows in an inviscid shock layer.

Fig.3 shows the dependence of the coefficients of friction τ_1, τ_2 and heat exchange coefficient q (lines *1–3* respectively) on the Reynolds number for $\beta = 1$ (solid lines) and $\beta = 8$ (dashed lines). Here $k = 0.1$; $\alpha = \delta = 0.5$; $\theta_w = 0.1$. The expressions for τ_α and q have the form

$$\tau_\alpha = \sqrt{\mathrm{Re}}\,\frac{\mu}{\rho_\infty V_\infty{}^2 u_\infty{}^\alpha}\,\frac{\partial u^\alpha}{\partial x^3},\quad q = \sqrt{\mathrm{Re}}\,\frac{\lambda}{\rho_\infty V_\infty{}^3}\,\frac{\partial T}{\partial x^3} \tag{2.6}$$

We see that even when the particle density in the oncoming flow in sufficiently low ($\delta = 0.5$), the nature of the interaction between the particles and the carrier phase has a strong influence on τ_α and q. In particular, the difference between the corresponding quantities can, for various values of β, be as high as 100% and the dependence of the heat exchange coefficient on the Reynolds number may be qualitatively quite different.

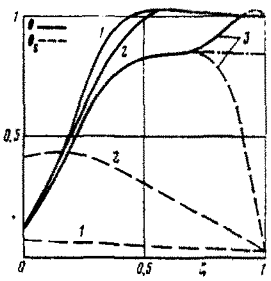

Fig.2

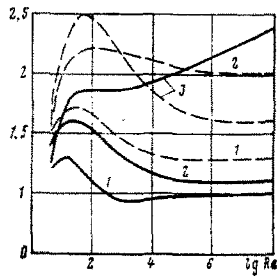

Fig.3

We also computed the flows in a two-phase viscous shock layer on a permeable surface. Here we considered values of the defining parameters, for which $v_{sw} < 0$. The analysis showed that in this case the nature of the flow within the shock layer is determined by the parameter $F_w = -G\sqrt{\mathrm{Re}}\cdot(1+k)^{-1/4}\varepsilon^{1/4}(-P_{1w})^{-1/4}$ which is normally used in the theory of a viscous shock layer in a homogeneous gas /5/. At fairly large values of $-F_w\,(-F_w > 3)$ inviscid flow occurs near the body (within the injection layer), and the boundary layer is displaced towards the shock layer and becomes the displacement layer. This is seen in Fig.1 where the lines 4 correspond to the profiles u, w, θ across the shock layer for $G = k = 0.1$; $\mathrm{Re} = 5\cdot 10^3$; $\alpha = 0.5$; $\beta = \delta = 1$. Note that the gas temperature relaxation layer is also displaced into the shock layer in the neighbourhood of the line $v = 0$.

3. Analysing (1.1)–(1.3) we find that the parameter β is the basic parameter determining the intensity of the phase interaction. It is proportional to the ratio of the characteristic gas-dynamic time to the characteristic gas and particle relaxation time. To explain the qualitative influence of the particles on the gas-dynamic characteristics of the flow, we shall analyse the asymptotic formulas for the initial equations (1.1) in two limiting cases, as $\beta \to 0$ and as $\beta \to \infty$. We shall study these cases in more detail.

When $\beta \to 0$, the solution of the resulting problem of regular perturbations is sought in the form of series in powers of β. Analysing the system of equations that holds for the principal terms of this expansion we find, that to a first approximation the motion of the condensed phase is independent of the gas, and the trajectories of the gas particles are straight lines parallel to the y^3 axis. Note that the reaction of the particles on the flow of carrier phase will be substantial to a first approximation only at a fairly high mass concentration of the particles, namely when $\rho_{s\infty} = O(\varepsilon\beta^{-1})$. If on the other hand $\rho_{s\infty} = O(1)$, then the above effect can be neglected.

When $\beta \to \infty$, analysis of (1.1) shows that to a first approximation the velocities and temperatures of the gas and the particles in the shock layer are the same. However, since the solution in question does not satisfy boundary conditions (1.2), a thin relaxation sublayer with thickness of the order of β^{-1} forms near the shock wave. In the coordinate system attached to the shock wave surface in the normal manner, the equations describing the flow in this layer, to a first approximation, have the form

$$\rho u^3 = u_\infty^3, \quad \varepsilon\rho_s u_s^3 = \delta u_\infty^3 \tag{3.1}$$

$$u_\infty^3 [(u^\alpha - u_\infty^\alpha) + \delta(u_s^\alpha - u_{s\infty}^\alpha)] = \frac{\mu\beta}{K}\frac{\partial u^\alpha}{\partial z}$$

$$u_\infty^3 \Big\{ T - \frac{T_\infty}{T_0} + \frac{1}{2}\Psi_{\alpha\beta}(u^\alpha - u_\infty^\alpha)(u^\beta - u_\infty^\beta) +$$

$$\delta\Big[\alpha^{-1}\Big(T_s - \frac{T_\infty}{T_0}\Big) + \frac{1}{2}\Psi_{\alpha\beta}(u_s^\alpha - u_\infty^\alpha)(u_s^\beta - u_\infty^\beta) -$$

$$\frac{1}{2}(\varepsilon u_s^3 - u_\infty^3)^2\Big] - \frac{1}{2}(\varepsilon u^3 - u_\infty^3)^2\Big\} + \varepsilon u^3\Big(\frac{2P}{1+\varepsilon} -$$

$$\frac{1}{\gamma M_\infty^2}\Big) = \frac{\mu\beta}{K\sigma}\frac{\partial T}{\partial z}, \quad u_s^3\frac{\partial u_s^i}{\partial z} = \mu\varphi(u^i - u_s^i)$$

$$u_\infty^3[\varepsilon u^3 - u_\infty^3 + \delta(\varepsilon u_s^3 - u_\infty^3)] = \frac{4}{3}\varepsilon\frac{\mu\beta}{K}\frac{\partial u^3}{\partial z} +$$

$$\frac{1}{\gamma M_\infty^2} - \frac{2P}{1+\varepsilon}$$

$$u_s^3\frac{\partial T_s}{\partial z} = \frac{2\,\mathrm{Nu}\,\mu\alpha}{3\sigma}(T - T_s), \quad z = \beta x^3$$

Note that the relations connnecting the limiting values of the parameters in the relaxation layer can be obtained, when $z\to\infty$, from (3.1) without considering the structure of this layer in detail. Denotng these parameters by the subscript g, we obtain the following finite relations:

$$u_{sg}^i = u_g^i, \quad T_{sg} = T_g, \quad u_\infty^3\delta_0(u_g^\alpha - u_\infty^\alpha) = \frac{\mu}{K}\frac{\partial u_g^\alpha}{\partial x^3} \tag{3.2}$$

$$u_\infty^3\Big\{\Big(1 + \frac{\delta}{\alpha}\Big)\Big(T_g - \frac{T_\infty}{T_0}\Big) + \frac{1}{2}\delta_0[\Psi_{\alpha\beta}(u_g^\alpha - u_\infty^\alpha)(u_g^\beta - u_\infty^\beta) -$$

$$(\varepsilon u_g^3 - u_\infty^3)^2]\Big\} + \varepsilon u_g^3\Big(\frac{2P_g}{1+\varepsilon} - \frac{1}{\gamma M_\infty^2}\Big) = \frac{\mu}{K\sigma}\frac{\partial T_g}{\partial x^3}$$

$$u_\infty^3\delta_0(\varepsilon u_g^3 - u_\infty^3) = \frac{4}{3}\frac{\mu}{K}\frac{\partial u_g^3}{\partial x^3} + \frac{1}{\gamma M_\infty^2} - \frac{2P_g}{1+\varepsilon}$$

$$\rho_g = \frac{P_g}{T_g}, \quad u_g^3 = \frac{u_\infty^3}{\rho_g}, \quad \rho_{sg} = \frac{\delta u_\infty^3}{\varepsilon u_g^3}, \quad \delta_0 = 1 + \delta$$

Following the method of asymptotic expansions /11/ we find, that outside the relaxation sublayer $u_s^i = u^i$, $T_s = T$, $\rho_s = \mathrm{const}\cdot\rho$. The equations which hold in this region are formally identical to a first approximation with the first five equations of (1.1), in which we must put $\beta = 0$, replace ρ in the first four equations by $\rho\delta_0$, and in the fifth equation by $\rho\cdot(1 + \delta/\alpha)$. The boundary conditions for these equations are identical at the body surface with conditions (1.3), and at the shock wave with conditions (3.2) written in the coordinate system attached to the body.

Near the stagnation point the above equations and boundary conditions (3.2) written in the hypersonic approximation in the variables (2.1)–(2.2), will take the form

$$\frac{\partial}{\partial\zeta}\Big(l\frac{\partial u^\alpha}{\partial\zeta}\Big) + \delta_0(f_1 + kf_2)\frac{\partial u^\alpha}{\partial\zeta} = \delta_0 d_{(\alpha)}(u^\alpha)^2 + \frac{2\varepsilon P_\alpha}{(1+\varepsilon)\rho} \tag{3.3}$$

$$\frac{\partial}{\partial\zeta}\Big(\frac{l}{\sigma}\frac{\partial\theta}{\partial\zeta}\Big) + \Big(1 + \frac{\delta}{\alpha}\Big)(f_1 + kf_2)\frac{\partial\theta}{\partial\zeta} = 0$$

$$\frac{\partial P_\alpha}{\partial\zeta} = \Delta\delta_0 d_{(\alpha)}^2(1+\varepsilon)(u^\alpha)^2, \quad \rho = \delta_0\frac{1+\varepsilon}{\theta}$$

$$\zeta = 1, \quad l\Delta\frac{\partial u^\alpha}{\partial\zeta} + \delta_0(u^\alpha - 1) = 0, \quad f_1 + kf_2 = \frac{1}{\Delta} \tag{3.4}$$

$$\frac{l\Delta}{\sigma}\frac{\partial\theta}{\partial\zeta} + \Big(1 + \frac{\delta}{\alpha}\Big)\theta = \delta_0, \quad P_\alpha = -d_\alpha\delta_0(1+\varepsilon)$$

The boundary conditions at the body surface and the same as (2.5).

Consider the asymptotic solution of (3.3), (3.4) and (2.5) at large Reynolds numbers. As in the case of a homogeneous gas /5/, the problem is singular as $K\to\infty$ and its asymptotic behaviour depends on the injection parameter. When $-F_w \leqslant 1$ the shock layer can be separated into the inviscid shock layer, and a boundary layer. When $-F_w \gg 1$, a three-layer model of the flow occurs, in which the effects of molecular transport can be neglected in the layers adjacent to the body and the shock wave, while in the intermediate region (displacement layer) the effect is of fundamental importance.

When solving the external problem we replace the displacement layer by a contact discontinuity with the corresponding conditions at this discontinuity /12/, and we assume the longitudinal pressure gradient $P_\alpha(\zeta)$ to be equal to $P_{\alpha w}$ where $P_{\alpha w}$ is given by the formulas obtained in the same manner as the Busemann-Hayes formula for a homogeneous gas /13/

$$P_{1w} = -\delta_0(1+\varepsilon)\left\{1 + \frac{1}{2(1-k)} - \frac{k}{(1-k)^2} - \frac{k^2 \ln k}{(1-k)^3}\right\} \tag{3.5}$$
$$P_{2w} = -\delta_0 k^2 (1+\varepsilon)\left\{\frac{1}{k} + \frac{k-3}{2(1-k)^2} - \frac{\ln k}{(1-k)^3}\right\}$$

From /12, 14/ it follows that such an approach is asymptotically correct at low values of the parameter ε. Taking into account (3.5), we write the solution of the external problem in the form

$$u^1 = \beta_{1i} t, \quad u^2 = \frac{\beta_{2i}}{\sqrt{k}} \frac{1 + C_{1i} t_*}{1 - C_{1i} t_*}, \quad v = f_1 + k f_2 = \frac{C_{2i} |t^2 - 1|^{1/2} t_*^{1/2}}{1 - C_{1i} t_*} \tag{3.6}$$
$$\zeta_1 = \beta_{11}^{-1} \int_0^t \frac{v\,dt}{t^2-1} \; (0 \leqslant t \leqslant 1), \quad \zeta_2 = \zeta_1(1) +$$
$$\beta_{12}^{-1} \int_1^t \frac{v\,dt}{t^2-1} \; (1 \leqslant t \leqslant \beta_{12}^{-1})$$
$$t_* = \left|\frac{t-1}{t+1}\right|^a, \quad a = \sqrt{k}\,\frac{\beta_{2i}}{\beta_{1i}}, \quad C_{11} = -1, \quad C_{21} = -\frac{2G}{\Delta \rho_w}$$
$$C_{12} = \frac{(1+\beta_{12})^a (\sqrt{k} - \beta_{22})}{(1-\beta_{12})^a (\sqrt{k} + \beta_{22})}, \quad C_{22} = \frac{2\beta_{12}\beta_{22}(1-\beta_{12})^{(a-1)/2}}{\Delta (1+\beta_{12})^{a/2} (\sqrt{k} + \beta_{22})}$$
$$\beta_{\alpha 2} = \left[-\frac{2\varepsilon P_{\alpha w}}{(1+\varepsilon)^2}\right]^{1/2} \left[\delta_0\left(1 + \frac{\delta}{\alpha}\right)\right]^{-1/2},$$
$$\beta_{\alpha 1} = \beta_{\alpha 2}\left[\theta_w\left(1 + \frac{\delta}{\alpha}\right)\right]^{1/2}$$

The index $i = 1, 2$ refers to the solution in the injection layer and shock layer respectively. The magnitude of the deviation Δ is found from the condition $\zeta_2(1/\beta_{12}) = 1$.

When the injection is intense $(-F_w \gg 1)$, the solution of the internal problem will consist of the solutions of boundary layer equations which are identical in the neighbourhood of the stagnation point with (3.3), provided that we put in them $\Delta = 1$, $P_\alpha = P_{\alpha w}$, $\partial P_\alpha / \partial \zeta = 0$ ($P_{\alpha w}$ is determined from the solution of the external problem at the contact surface). The boundary conditions are

$$\zeta \to +\infty: \theta \to \frac{\delta_0}{(1+\alpha^{-1}\delta)}, \quad u^\alpha \to \left[-\frac{2\varepsilon P_{\alpha w}}{d_{(\alpha)}(1+\varepsilon)^2}\right]^{1/2} \times \left[\delta_0\left(1 + \frac{\delta}{\alpha}\right)\right]^{-1/2} \tag{3.7}$$
$$\zeta \to -\infty: \theta \to \theta_w, \quad u^\alpha \to \left[-\frac{2\varepsilon P_{\alpha w}\theta_w}{d_{(\alpha)}(1+\varepsilon)^2 \delta_0}\right]^{1/2}$$
$$\zeta = \zeta_c, \quad f_1 + k f_2 = 0$$

When the injection is weak $(-F_w \leqslant 1)$ the internal problem also consists of solving the boundary layer equations. The boundary conditions at the body surface are given by (2.5), and at the outer boundary by the first condition of (3.7). The system of equations (3.3) with boundary conditions (2.5), (3.4) describing the equilibrium flow of a gas containing particles (the limiting case of fine particles), was also solved by numerical methods.

Figs.4 and 5 show the dependence of the heat exchange coefficient q at the impermeable

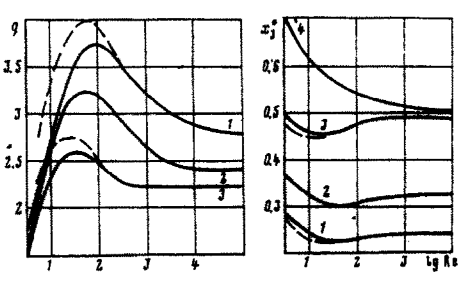

Fig.4

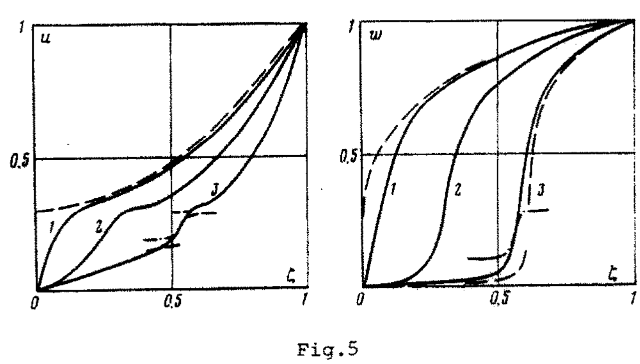

Fig.5

surface and the separation of the shock wave x_3^* on the Reynolds number for various values of the principal radii of curvature of the body at the stagnation point. Here $\delta = 1$; $\alpha = 0.5$; $\theta_w = 0.1$; $k = 1$; 0.5; 0.1 are the lines *1–3* respectively and $\theta_w = 0.5$; $k = 0.1$ is line *4*. The dashed lines depict the computations without slippage and temperature jump at the body surface. We see that when $Re \leqslant 50$, then disregarding the slippage and temperature jump causes a 15—20% error in determining q.

Computations have shown that in the case of small k the values of the coefficient of friction τ_2 computed from the solution of the equations of the viscous shock layer and the boundary layer show considerable deviations from each other even at large Reynolds numbers, while the differences in the values of the coefficients τ_1 and q are practically nil at $Re \geqslant 5 \cdot 10^5$. This is due to the fact that as $k \to 0$, the value of $\partial u^2/\partial\zeta|_{\zeta=0}$ obtained from the solution of the external problem tends to infinity by virtue of the fact that the correct construction of the asymptotic forms of the boundary value problem (3.3), (2.5), (3.4) with $Re \to \infty$, must take due regard to the vortex interaction to a first approximation. This result was obtained earlier in /5/ for a flow of homogeneous gas.

It is interesting to note that the nature of the dependence of the separation of the shock wave on the Reynolds number is strongly influenced by the surface temperature. When the wall is cold $(\theta_w \leqslant 0.25)$, the dependence in not monotonic and has a local minimum, while at fairly high wall temperatures $(\theta_w \geqslant 0.4)$ the separation decreases montonically as the Reynolds number increases.

We have also computed the two-phase equilibrium flow in a three-dimensional hypersonic viscous shock layer with injection. Fig.5 shows the velocity profiles u and w across the shock layer at $\delta = 1$; $\alpha = 0.5$; $\theta_w = 0.1$; $Re = 5 \cdot 10^3$ where 5 $(G = 0; 0.1; 0.25$ correspond to lines *1—3*. The dashed line depicts the analytical solution (3.6) of the external problem, and the dot-dash line the numerical solution of the internal problem (3.3), (3.7). It is clear that while good agreement is obtained between the numerical and asymptotic solutions for the profiles of u, a much larger discrepancy occurs between those solutions for the profiles of w. This can be explained by the fact that in the case of intense injection a vortex layer is already formed near the surface of contact discontinuity.

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